My current research interests are in the intersection of functional analysis and probability. My most recent work as been the study of small deviations for Brownian motion with independent time-change. I have also the studied stochastic processes on Lie groups and proved results for processes on infinite-dimensional analogues of the Heisenberg group.

In [12] we prove small deviation estimates for some time-changed Brownian motions, for the purpose of applications to certain elements of Wiener chaos. Large deviation estimates for Wiener chaos are well-studied (see for example [23]), largely due to the work of Borell (see for example [8] and [9]). However, small deviations in this setting are much less understood and are of interest for their myriad interactions with other concentration, comparison, and correlation inequalities as well as various limit laws for stochastic processes; see for example the surveys [24] and [25]. To my knowledge, the results presented [12] are the first for small deviations of elements of Wiener chaos in the infinite-dimensional context beyond the first-order Gaussian case.

Another area that I have studied is the smoothness properties of the law of Brownian motion on such groups. In finite dimensions, we usually say a measure is smooth if it is absolutely continuous with respect to a reference measure and has a smooth density. Since there is no canonical reference measure in infinite dimensions, weaker notions of smoothness, such as those given in [5, 26, 27], are usually used. In the paper [11], we build on the results of [15] and show that the law of Brownian motion on an infinite-dimensional Heisenberg-like group satisfies a strong definition of smoothness.

Background

Recall that the Heisenberg group may be realized as $\mathbb{H} = \mathbb{R}^2 \times \mathbb{R}$ with group operation given by

$$(x_1, x_2, x_2) \cdot (y_1, y_2, y_3) = \left(x_1 + y_1, x_2 + y_2, x_3 + y_3 + \frac{1}{2}(x_1 y_2 - x_2 y_1) \right).$$

Let (B_t^1, B_t^2, B_t^0) be a Brownian motion on $\mathbb{R}^3$. Then

$$A(t) = \frac{1}{2} \int_0^t B_s^1 dB_s^2 - B_s^2 dB_s^1 \tag{1}$$

is a stochastic Lévy area process and $(B_t^1, B_t^2, B_t^0 + A_t)$ is a Brownian motion on $\mathbb{H}$. An infinite-dimensional Heisenberg-like group G is an analogue of $\mathbb{H}$ where $\mathbb{R}^2$ is replaced with a separable Banach space W equipped with a Gaussian measure μ and $\mathbb{R}$ is replaced with a finite-dimensional Hilbert space $\mathbf{C}$.

The infinite-dimensional Heisenberg-like groups I study were first introduced in [15]. Roughly stated, given an abstract Wiener space (W, H, μ) and a finite-dimensional Hilbert space $\mathbf{C}$, an infinite-dimensional Heisenberg-like group is the group $G = W \times \mathbf{C}$ with group operation

$$(w_1, c_1) \cdot (w_2, c_2) := \left(w_1 + w_2, c_1 + c_2 + \frac{1}{2}\omega(w_1, w_2) \right)$$

where $(w_j, c_j) \in G$ for $j = 1, 2$ and $\omega : W \times W \to \mathbf{C}$ is an anti-symmetric bilinear operator.

It should be noted that $W \times \mathbf{C}$ can be thought of as both a (Lie) group and as a Lie algebra with respect to the bracket $[g_1, g_2] := (0, 0, \omega(g_1, g_2))$. When thought of as a group, we will denote $W \times C$ as G and when thought of a Lie algebra, $\mathfrak{g}$.

Given a Brownian motion (B_t, B_t^0) on $(W, \mathbf{C})$, Brownian motion on G is $\{\xi_t\}_{t \geq 0}$ where

$$\xi_t = \left(B_t, B_t^0 + \frac{1}{2} \int_0^t \omega(B_s, dB_s) \right). \tag{2}$$

We denote by ν_T the time T heat kernel measure, that is, the law of ξ_T. Among other things, the authors of [15] show that the law of ξ on the path space of G is quasi-invariant with respect to left multiplication by finite energy paths and that ν_T is quasi-invariant with respect to both left and right multiplication by elements of an appropriate Cameron-Martin subgroup of G.

Other properties of heat kernel measures on infinite-dimensional Heisenberg-like groups have been studied in [4, 18, 16]. The paper [16] extends the work in [15] by proving Taylor, skeleton, and holomorphic chaos isomorphism theorems for infinite-dimensional Heisenberg-groups built on complex abstract Wiener spaces. In [4] it is shown that heat kernel measures on sub-Riemannian infinite-dimensional Heisenberg-like groups are quasi-invariant and in [18] a Taylor isomorphism theorem is proven for heat-kernel measures on sub-elliptic infinite-dimensional Heisenberg-like groups.

Define $Z(t) := \frac{1}{2}\int_0^t \omega(B_s, dB_s)$ as in equation (2). In some sense this is an anologue of the Lévy area process and under the proper assumption on ω can be shown to be equal in distribution to a time-changed Brownian motion.

First-order small deviation estimates of the standard form

$$\log \mathbb{P}\left(\sup_{0\le s\le t} |Z(s)| \le \varepsilon\right)$$

were studied in [28] for processes $Z(t) = \int_0^t \omega(W_s, dW_s)$ with W an n-dimensional Brownian motion and $\omega : \mathbb{R}^n \to \mathbb{R}$ given by $\omega(x, y) = Ax \cdot y$ for A a skew-symmetric $n \times n$ matrix. These estimates were then applied to prove an analogue of the classical limit result of Chung. (This was done earlier in [29] in the case $n = 2$ and $A = \begin{pmatrix} 0 & 1/2 \\ -1/2 & 0 \end{pmatrix}$, that is, for Z the stochastic Lévy area.) In [21], the authors improved these results by proving stronger asymptotic results for the same Z as in [28] and applying these results to prove a functional law of iterated logarithm.

Recent Work

Smoothness

In [11], we show that both the path space measure and ν_T satisfy an appropriate adaptation of the following definition of a smooth measure given in Theorem 3.3 of [14]: A measure μ is smooth if for any multi-index $\alpha = (\alpha_1, \dots, \alpha_n) \in \{0, 1, 2, \dots\}^n$, there exists a function $z_\alpha \in C^\infty(\mathbb{R}^n) \cap L^{\infty-}(\mu)$ such that

$$\int_{\mathbb{R}^n} \partial^\alpha f \, d\mu = \int_{\mathbb{R}^n} f z_\alpha \, d\mu, \quad \text{for all } f \in C_c^\infty(\mathbb{R}^n),$$

where $L^{\infty-} := \cap_{p\ge1} L^p$ and $\partial^\alpha = \prod_{i=1}^n \partial_i^{\alpha_i}$.

Specifically, we adapt the above definition of smoothness to the group setting and give a direct proof of the smoothness of elliptic heat kernel measures on infinite-dimensional Heisenberg-like groups. In particular, let G be an infinite-dimensional Heisenberg-like group, $\mathfrak{g}_{CM}$ be its Cameron-Martin Lie subalgebra, and $\{\xi_t\}_{t\ge0}$ be a Brownian motion on G. Then we have the following theorem.

Theorem 1. *Fix $T > 0$, and let $m \in \mathbb{N}$ and $h_1, \dots, h_m \in \mathfrak{g}_{CM}$. Then there exist $\tilde{z}, \hat{z} \in L^{\infty-}$ depending on $h_1, \dots, h_m$ such that, for any suitably nice function f on G,*

$$\mathbb{E}\left[(\tilde{h}_1 \cdots \tilde{h}_m f)(\xi_T)\right] = \mathbb{E}[f(\xi_T)\tilde{z}] \quad \text{and} \quad \mathbb{E}\left[(\hat{h}_1 \cdots \hat{h}_m f)(\xi_T)\right] = \mathbb{E}[f(\xi_T)\hat{z}],$$

where $\tilde{h}$ and $\hat{h}$ are the left and right invariant vector fields, respectively, associated to $h \in \mathfrak{g}_{CM}$.

This result is proved by first establishing smoothness results for the induced measure on the associated path space. In particular, let $\mathcal{W}_T(G)$ denote continuous path space on G and $\mathcal{H}_T(\mathfrak{g}_{CM})$ denote the space of absolutely continuous paths on $\mathfrak{g}_{CM}$ with finite energy. Then we prove the following theorem.

Theorem 2. *Let $m \in \mathbb{N}$ and $\mathbf{h}_1, \dots, \mathbf{h}_m \in \mathcal{H}_T(\mathfrak{g}_{CM})$. Then there exists $\hat{Z} \in L^{\infty-}$ depending on $\mathbf{h}_1, \dots, \mathbf{h}_m$ such that, for any suitably nice function F on $\mathcal{W}_T(G)$,*

$$\mathbb{E}\left[(\hat{\mathbf{h}}_1 \cdots \hat{\mathbf{h}}_m F)(\xi)\right] = \mathbb{E}[F(\xi)\hat{Z}],$$

where $\hat{\mathbf{h}}$ is the right invariant vector field associated to $\mathbf{h} \in \mathcal{H}_T(\mathfrak{g}_{CM})$.

Typically, it is not possible to verify that a measure on an infinite-dimensional space is smooth in this way and much weaker interpretations must be made; see for example [5, 14, 26, 27]. Other references to quasi-invariance and integration by parts results for measures in infinite-dimensional curved settings include [1, 2, 6, 13, 17, 19] and their references.

Small Deviations

In the work [12] we work in the context of the following assumptions to give strong small deviations results for certain elements of second-order homogeneous chaos.

In particular, let $(\mathcal{W}, \mathcal{H}, \mu)$ be an abstract Wiener space, $\{W_t\}_{t\geq 0}$ denote Brownian motion on $\mathcal{W}$, and ω be a continuous bilinear antisymmetric map on $\mathcal{W}$. We study processes $\{Z(t)\}_{t\geq 0}$ of the form

$$Z(t) := \int_0^t \omega(W_s, dW_s). \tag{3}$$

(For a precise definition of Z, see [12, 15].)

Our main result the Theorem 4 which is small deviations result for time-changed Brownian with random clocks satisfying certain assumptions.

Assumption 3. Suppose $\{C(t)\}_{t\geq 0}$ is a continuous non-negative non-decreasing process such that $C(0) = 0$ and there exist $\alpha > 0$, $\beta \in \mathbb{R}$, and a non-decreasing function $K : (0, \infty) \to (0, \infty)$ such that for any $m \in \mathbb{N}$, $\{d_i\}_{i=1}^m \subset (0, \infty)$ a decreasing sequence, and $0 = t_0 < t_1 < \cdots < t_m$,

$$\lim_{\varepsilon \downarrow 0} \varepsilon^\alpha |\log \varepsilon|^\beta \log \mathbb{P}\left(\sum_{i=1}^m d_i \Delta_i C \leq \varepsilon\right) = -\left(\sum_{i=1}^m (d_i^\alpha K(t_{i-1}, t_i))^{1/(1+\alpha)}\right)^{(1+\alpha)} \tag{4}$$

where $\Delta_i C = C_{t_i} - C_{t_{i-1}}$.

Note that it is not necessary for C to have independent or stationary increments for Assumption 3 to hold. Working under this assumption one may prove the following.

Theorem 4. *Suppose that $\{Z(t)\}_{t\geq 0}$ is a stochastic process given by $Z(t) = B(C(t))$, where C satisfies certain assumptions and B is a standard real-valued Brownian motion independent of C. Let $M(t) := \sup_{0\leq s\leq t} |Z(s)|$. Then, for any $m \in \mathbb{N}$, $0 = t_0 < t_1 < \cdots < t_m < \infty$, and $0 \leq a_1 < b_1 \leq a_2 < b_2 \leq \cdots \leq a_m < b_m$,*

$$\lim_{\varepsilon \downarrow 0} \varepsilon^{2\alpha/(1+\alpha)} |\log \varepsilon|^{\beta/(1+\alpha)} \log \mathbb{P}\left(\bigcap_{i=1}^m \{a_i \varepsilon \leq M(t_i) \leq b_i \varepsilon\}\right)$$
$$= -2^{-\beta/(1+\alpha)}(1+\alpha)^{1+\beta/(1+\alpha)} \left(\frac{\pi^2}{8\alpha}\right)^{\alpha/(1+\alpha)} \sum_{i=1}^m \left(\frac{K(t_{i-1}, t_i)}{b_i^{2\alpha}}\right)^{1/(1+\alpha)}.$$

Such estimates have been previously studied for processes $\{Z_t\}_{t\geq 0}$ that are symmetric α-stable processes [10], fractional Brownian motions [20], certain stochastic integrals [21], m-fold integrated Brownian motions [32], and integrated α-stable processes [31]. In particular, the stochastic integrals studied in [21] are essentially finite-dimensional versions of the class of stochastic integral processes we study, and the proof that we give for Theorem 4 follows the outline of the proof of small ball estimates in that reference.

Our result for stochastic integrals of the form (3) is as follows.

Theorem 5. *Let $\{Z(t)\}_{t\geq 0}$ be as in equation (3). Then $\{Z(t)\} \stackrel{d}{=} \{B(C(t))\}$ for B a standard real-valued Brownian motion and*

$$C(t) = \sum_{k=1}^{\infty} \|\omega(e_k,\cdot)\|^2_{\mathcal{H}^*} \int_0^t (W_s^k)^2\, ds$$

where $\{e_k\}_{k=1}^{\infty}$ is any orthonormal basis of $\mathcal{H}$ contained in $\mathcal{H}_* := \{h \in \mathcal{H} : \langle h,\cdot\rangle$ extends to a continuous linear functional on $\mathcal{W}\}$ and $\{W^k\}_{k=1}^{\infty}$ are independent standard Brownian motions which are also independent of B. If we further suppose that $\|\omega(e_k,\cdot)\|_{\mathcal{H}^*} = O(k^{-r})$ for $r > 1$ or $\|\omega(e_k,\cdot)\|_{\mathcal{H}^*} = O(\sigma^k)$ for $\sigma \in (0,1)$, then, for any $m \in \mathbb{N}$, $0 = t_0 < t_1 < \cdots < t_m$, and $\{d_i\}_{i=1}^m \subset (0,\infty)$ a decreasing sequence,

$$\lim_{\varepsilon\downarrow 0} \varepsilon \log \mathbb{P}\left(\sum_{i=1}^m d_i^2 \Delta_i C \leq \varepsilon\right) = -\frac{1}{2}\|\omega\|_1^2 \left(\sum_{i=1}^m d_i \Delta_i t\right)^2,$$

where

$$\|\omega\|_1 := \sum_{k=1}^{\infty} \|\omega(e_k,\cdot)\|_{\mathcal{H}^*} < \infty.$$

Thus, for any $0 \leq a_1 < b_1 \leq a_2 < b_2 \leq \cdots \leq a_m < b_m$,

$$\lim_{\varepsilon\downarrow 0} \varepsilon \log P\left(\bigcap_{i=1}^m \{a_i\varepsilon \leq \sup_{0\leq s\leq t_i} |Z_s| \leq b_i\varepsilon\}\right) = -\frac{\pi}{4}\|\omega\|_1 \sum_{i=1}^m \frac{\Delta_i t}{b_i}.$$

Applications of such estimates include using the small deviations in Theorem 5 to prove a Chung-type law of iterated logarithm as well as a functional law of iterated logarithm for the process Z.

Future Work

It is my goal to establish a research program. The following are descriptions of some of problems I am currently thinking about.

Complete Polynomials

Recall (see for example [7, 22]) the integration by parts formula for an abstract Wiener space (W, H, μ) following from the standard Cameron-Martin theorem. Let $\{e_i\}_{i=1}^{\infty}$ be an orthonormal basis of H, and let ∂_i denote the derivative in the direction e_i. Then, for any $k \in \mathbb{N}$, distinct indices $i_1, \ldots, i_k$, and multi-index $\alpha = (\alpha_1, \ldots, \alpha_k) \in \mathbb{N}^k$, we have

$$\int_W (\partial_{i_1}^{\alpha_1} \cdots \partial_{i_k}^{\alpha_k} f)(w)\, d\mu(w) = \int_W f(w) H^{\alpha}_{i_1,\ldots,i_k}(w)\, d\mu(w)$$

for $H^{\alpha}_{i_1,\dots,i_k}(w) := \prod_{j=1}^k H_{\alpha_j}(\langle e_{i_j}, w\rangle_H)$, where H_n are the usual Hermite polynomials and "$\langle e_i, w\rangle_H$" is the Paley-Wiener integral.

On the other hand, Theorem 2 implies that, for all $\mathbf{h}_1,\dots,\mathbf{h}_m \in \mathcal{H}_T(\mathfrak{g}_{CM})$, there exists $\hat{\Phi}_{\mathbf{h}_1,\dots,\mathbf{h}_m} \in L^{\infty-}$ such that

$$\int_{\mathcal{W}_T(G)} (\hat{\mathbf{h}}_1\cdots\hat{\mathbf{h}}_m F)(\omega)\, d\nu(\omega) = \mathbb{E}\left[(\hat{\mathbf{h}}_1\cdots\hat{\mathbf{h}}_m F)(\xi)\right] = \mathbb{E}\left[F(\xi)\hat{\Phi}_{\mathbf{h}_1,\dots,\mathbf{h}_m}(\xi)\right] = \int_{\mathcal{W}_T(G)} F(\omega)\hat{\Phi}_{\mathbf{h}_1,\dots,\mathbf{h}_m}(\omega)\, d\nu(\omega).$$

In particular, $\hat{\Phi}_{\mathbf{h}_1,\dots,\mathbf{h}_m}(\xi) = \mathbb{E}[\Phi_{\mathbf{h}_1,\dots,\mathbf{h}_m}|\sigma(\xi_t, t\in[0,T])]$ a.s., and comparing this with the above flat case leads one to think of Φ as a polynomial of order m in

$$\langle \mathbf{h}_i, (B, B^0)\rangle_{\mathcal{H}_T(\mathfrak{g}_{CM})} := \int_0^T \langle \dot{\mathbf{h}}_i(t), d(B_t, B_t^0)\rangle_{\mathfrak{g}_{CM}} = \int_0^T \langle \dot{\mathbf{A}}_i(t), dB_t\rangle_H + \int_0^T \langle \dot{\mathbf{a}}_i(t), dB_t^0\rangle_{\mathbf{C}}$$

as well as additional terms like $\int_0^T \langle \omega(B, \dot{\mathbf{A}}_i), dB^0\rangle_{\mathbf{C}}$. The presence of these additional terms of course follows from the non-commutativity of the setting. That is, our formula coincides with the flat case in the event that $\omega \equiv 0$ and motivates the following.

Question 1. What is the relationship between polynomials Φ and the Hermite polynomials? What properties does Φ maintain in the non-commutative setting that is has in the flat case? Is the collection still orthogonal, complete, etc.

Hermite polynomials on the Heisenberg group are studied in [30] and I hope to prove or disprove similar results for the polynomials Φ in above.

Applications of Small Deviations

Recall that a process X is H-self-similar if $\{X(ct)\} \stackrel{d}{=} \{c^H X(t)\}$ for all $c > 0$. For example, fractional Brownian motion with Hurst parameter H is H-self-similar and α-stable Lévy processes are $1/\alpha$-self-similar.

Consider a process of the form $B(Y(t))$ where $Y(t)$ is a continuous H-self-similar process independent of a two-sided Brownian motin B such that Y has stationary increments, $Y(0) = 0$ almost surely and

$$\lim_{\varepsilon\downarrow 0} \varepsilon^{\tau} \log \mathbb{P}\left[\sup_{s,t\in[0,1]} |Y(s) - Y(t)| \le \varepsilon\right] = -k \tag{5}$$

for some $\tau > 0$.

The following is a special case of Theorem 4 in [3] which gives small deviations results for $B(Y(t))$ where B is a two-sided Brownian motion and Y is as above.

Theorem 6. *Let B be a two-sided Brownian motion and Y a continuous process independent of B such that $Y(0) = 0$ almost surely and (5) holds. Then*

$$\lim_{\varepsilon\downarrow 0} \varepsilon^{\frac{2\tau}{1+\tau}} \log \mathbb{P}\left[\sup_{0\le t\le 1} |B(Y(t))| \le \varepsilon\right] = -\left(\frac{\pi^2}{8}\right)^{\frac{\tau}{1+\tau}} k^{\frac{1}{1+\tau}} \tau^{-\frac{\tau}{1+\tau}} (1+\tau).$$

Remark 7. Theorem 6 also holds of Y satisfies the Anderson property (that is, $\mathbb{P}(Y \in A) \geq \mathbb{P}(Y \in A + x)$ for any $x \in E$ where E is a convex set containing A) and

$$\lim_{\varepsilon \downarrow 0} \varepsilon^{\tau} \log \mathbb{P}\left[\sup_{0 \leq t \leq 1} |Y_t| \leq \varepsilon\right] = k2^{-\tau}.$$

See, for example, Lemma 7 of [3].

Small deviations estimates like those in Theorem 6 are often used to prove limit laws. I conjecture the following Chung-like law of the iterated logarithm for $B(Y(t))$.

Theorem 8. *Let B be a real-valued two-sided Brownian motion and Y a continuous H-self-similar process independent of B such that Y has stationary increments, $Y(0) = 0$ almost surely and*

$$\lim_{\varepsilon \downarrow 0} \varepsilon^{\tau} \log \mathbb{P}\left[\sup_{s,t \in [0,1]} |Y(s) - Y(t)| \leq \varepsilon\right] = -k.$$

Then,

$$\mathbb{P}\left[\liminf_{T \to \infty} T^{-H/2} (\log \log T)^{(1+\tau)/2\tau} \sup_{0 \leq s \leq T} |B(Y(s))| = \left(\frac{\pi^2}{8}\right)^{\frac{\tau}{1+\tau}} k^{\frac{1}{1+\tau}} \tau^{-\frac{\tau}{1+\tau}} (1 + \tau)\right] = 1.$$

To this point, I have proven the lower bound and am endeavoring to prove the upper bound.

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